Elastic-Inelastic Power Function Model

Elastic-inelastic power function model was first created by Mr. Baoquan An to simulate rock impact process in his Master thesis titled “A Study of Energy Loss During Rock Impact Using PFC2D” in 2006. This user-defined constitutive model (UDM) incorporates an energy dissipation algorithm. Detailed theoretic analyses of the dissipation algorithm were given in this thesis as well.

The elastic-inelastic power function model consists of a linear elastic process, an inelastic process and a power function process. This model can be used to simulate the energy loss due to collisions, such as rockfall.

1 Elastic-Inelastic Power Function Model

Elastic-inelastic power function model is a complex normal contact model, which includes a linear elastic process, an inelastic process and a power function dumping process, shown in Figure 1. During the linear elastic process *OA*, normal contact force between two particles increases in proportion to relative normal displacement; during the inelastic process *AB*, the normal contact force doesn’t change while the normal displacement increases; during the power function dumping process *BO*, the normal contact force decreases with the decreased normal displacement. Transition point *B*, at which the model transfers from loading process to unloading process, is automatically determined by the contact.

The elastic-inelastic power function model may work together with a default frictional slip model and shear model provided by PFC, which are not presented here. The elastic-inelastic power function model has the following parameters:

Transition force	To specify transition force (force), at which the model transfers from elastic process *OA* to inelastic process *AB*
Exponent	To specify exponent b for the power function $y = ax^b$

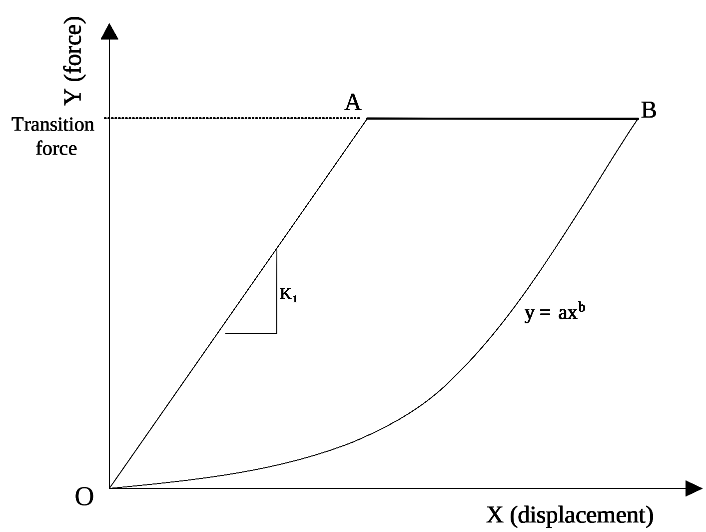

Figure 1 Elastic-inelastic power function model

The parameters used in the elastic-inelastic power function model are *Transition force* and *Exponent*. *Transition force* is used to control the normal contact force level, at which a linear elastic force-displacement relationship transfers to an inelastic process. *Exponent* determines the power for damping function. The value of exponent, b has a dominant control on the energy loss during impact and hence can be considered to define restitution coefficient.

If *transition force* is given a very large value so that the inelastic process is prevented, when two contacting particles begin to move away from each other, the normal force will follow the power function defined by $y = ax^b$. The elastic-inelastic power function model shown in Figure 1 then becomes an elastic-power function model shown in Figure 2 (a). If the exponent of the power function is set to 1.0, the elastic-inelastic power function model becomes a triangular damping model shown in Figure 2 (b).

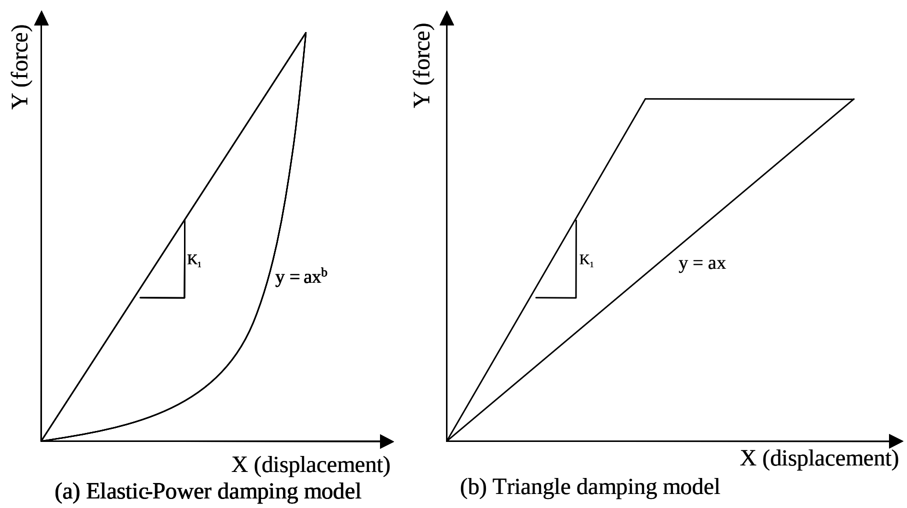

Figure 2 Different forms of elastic-inelastic power function model

2 Damping and Normal Restitution Coefficient

The schematic plot of the energy dissipation Algorithm of the elastic-inelastic power function model is shown in Figure 3.

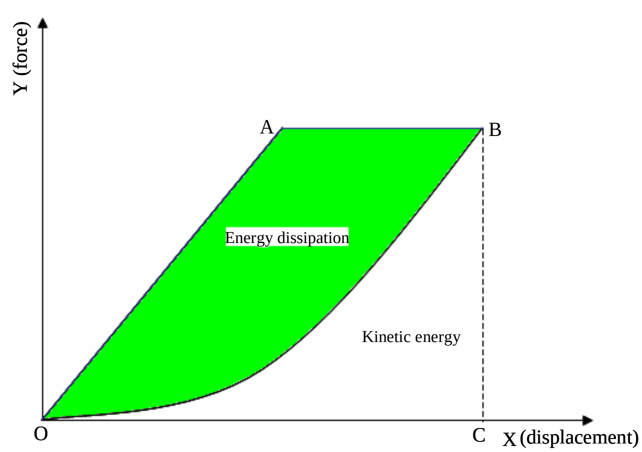

Figure 3 Schematic of energy dissipation of elastic-inelastic power function model

In the Figure 3, the area of *OABC* represents the total work that the contact force does during the impact process. Since the contact force follows the power function from *B* to *O* during the bounce process, the work transferred into kinetic energy after bounce is represented by the area of *OBC*. The energy represented by the area *OAB* is removed by the elastic-inelastic power function model during one impact-bounce cycle.

By using different power functions, the elastic-inelastic power function model can give various damping results. Therefore, the power function ($y = ax^b$) actually defines the damping results. For instance, the larger the exponent *b*, the less kinetic energy available after the bounce. The ratio of the areas *OBC* and *OABC* can be used to specify normal restitution coefficient:

$$Normal\ restitution\ coefficient = \sqrt{\frac{Area(OBC)}{Area(OABC)}} \qquad \text{Equation 1}$$

3 Relationship between Normal Restitution Coefficient and UDM

A general relationship between the *exponent* and normal restitution coefficient can be established based on mathematic analysis. Normal restitution coefficient achieved by the elastic-inelastic power function model can be expressed as

$$Normal\ restitution\ coefficient = \sqrt{\frac{2}{b+1}} \times \sqrt{\frac{g+1}{g+2}} \qquad \text{Equation 2}$$

where, *b* is the exponent of the power function and *g* is the ratio of elastic deformation to inelastic deformation. For derivation of the above equation, please refer to Mr. Baoquan An's thesis.

If *g* is very large or infinite, the second term in Equation 2 will be close to unity, and the normal restitution coefficient will be determined by *b* only. This actually occurs when the *transition force* is specified as a very large value so that the elastic-inelastic power function model transfers to an elastic power function model. For elastic power function model, normal restitution that can be achieved is:

$$Normal\ restitution\ coefficient = \sqrt{\frac{2}{b+1}} \qquad \text{Equation 3}$$

The damping results of the elastic-inelastic power function model are shown in Figure 4.

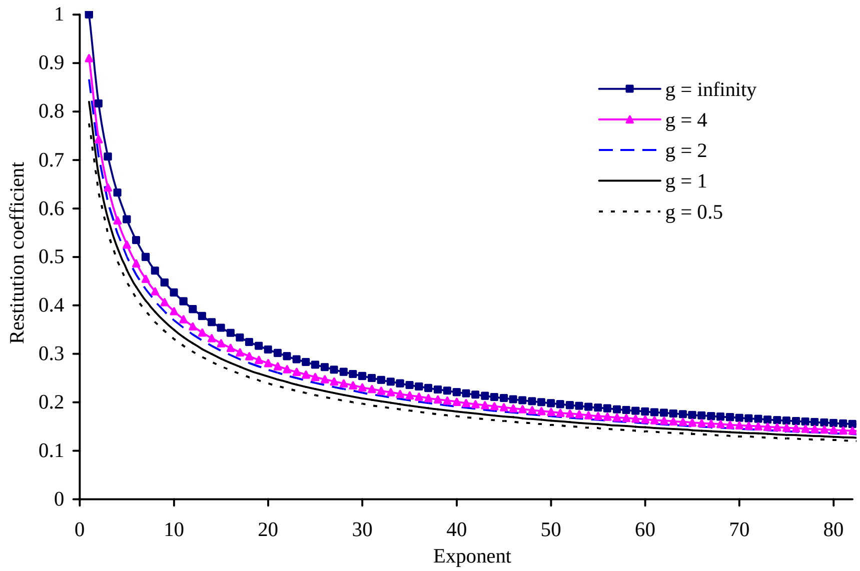

Figure 4 Normal restitution coefficient vs. exponent for various values of *g*
(Upper curve represents damping results of elastic power function model)

4 Dynamic Link Library and Example

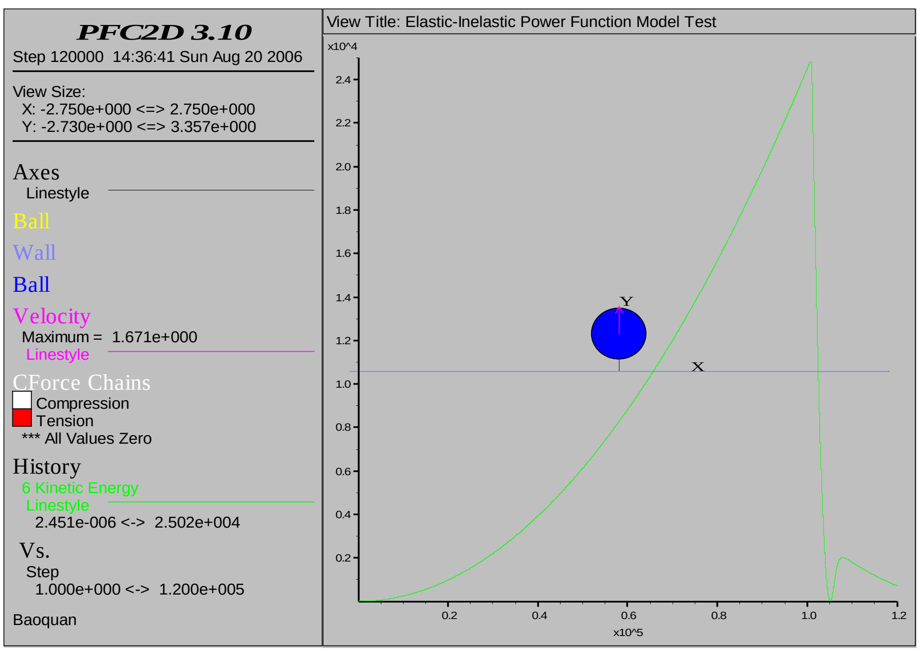

Figure 5 Restitution of coefficient is 0.28 when Transition force=128000 and Exponent =10

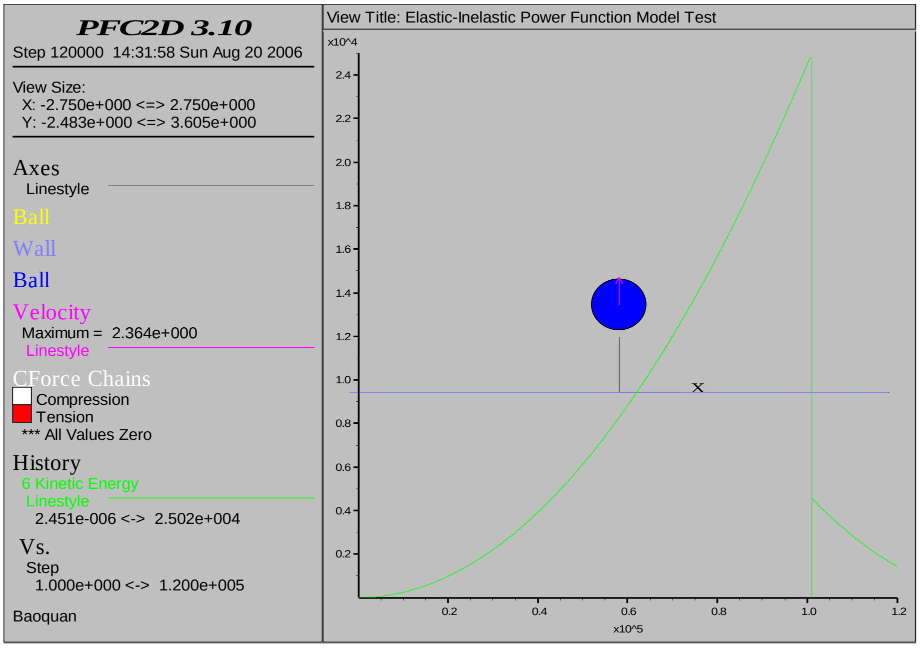

Figure 6 Restitution of coefficient is about 0.42 when Transition force=128000000 and Exponent =10

An example program and DLL file containing the elastic-inelastic power function model are provided to demonstrate how to use the new contact model. The damping results of two cases are show in Figure 5 and Figure 6. Figure 6 actually shows a case of elastic power function model. The user may compare this result with Figure 4 or Equation 3. Baoquan An's thesis titled "A Study of Energy Loss During Rock Impact Using PFC2D" is attached for reference.

Because the time to construct and implement the elastic-inelastic power function model was very limited, this contact model might contain some bugs. Any suggestions will be welcome.

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; ==============================================================
;=============== Elastic-Inelastic Power Function Model ==================
; ==============================================================
;This program is used to check the damping result of Elastic-Inelastic Power Function
;Model.The users can trace energy kinetic to observe damping results;Attentation should
;be paid to timestep. The default value might not work well, therefore ;the user should set
;new timestep and total cycles in such a case.
;The restitution of coefficient is about 0.28 when set_Transition_force=128000 and
;setExponent =10. The restitution of coefficient is about 0.42 when set_Transition_force
;=128000000 and setExponent =10; in latter case, The Elastic-Inelastic Power Function
;Model will be transferred to Elastic- Power Function Model if the Transition force is given
;a large value. The restitution of coefficient achieved is consistent with theoretic results
;(refer to Figure 4)
;Two key parameters needed to be specified by the used for the model are:
;		Transition_Force -- to define transition force for the model
;		Exponent -- to define exponent for the power function of the model
;===============================================================
new
set dt max 1.0e-5
set disk on
trace energy on
set pinterval 50
set damp local 0.0
set hist_rep 1
config cppudm
model load Elastic_Inelastic_Power_Function_Model.dll;
; To load the file containing Elastic_Inelastic_Power_Function model
;pause key
;--------------------------------------------------------------------
def make_ball
command
ball id =1 rad 0.25 x = 0.0 y = 5.251
prop dens = 2600 kn 3.2e10 ks 9.6e9 fric 0.6
end_command
end
;--------------------------------------------------------------------
def zero_damp
bp = ball_head
loop while bp # null
b_damp(bp) = 0.0
bp = b_next(bp)
end_loop
end
;--------------------------------------------------------------------
def catch_contact_model
cp = fc_arg(0);
c_model(cp) = 'Elastic_Inelastic_Power_Function'
c_prop(cp,'Transition_Force')= set_Transition_force ; to specify transition force
c_prop(cp,'default_ks')=9.0e6     ; shear stiffness for default contact model
```

```
c_prop(cp,'default_fric')=0.6     ; friction coefficient for default frictional slip model
c_prop(cp,'Exponent')= setExponent ; to specify exponent for the power function
c_prop(cp,'model_notension')= 1
 ; always specified as 1, might be used for extension of the model
c_prop(cp,'model_inheritprop')= 1
 ; always specified as 1, might be used for extension of the model
end
;------------------------------------------------------------------
def plot_view
command
pl create One_ball_drops
pl set title text "Elastic-Inelastic Power Function Model Test"
plot add axes black
plot add ball yellow
plot add wall lblue
pl add ball vel ma
plot add cf white
hist id 6 energy kinetic
pl add hist 6 green
end_command
end
;------------------------------------------------------------------
wall id = 1 kn 6.4e10 ks 9.6e9 fric 0.6 &
nodes (-2.5,0.0) (2.5,0.0)
set grav 0 -9.8
plot show
make_ball
zero_damp
plot_view
;------------------------------------------------------------------
set set_Transition_force =128000; To specify transition force for the model
set setExponent = 10; To define exponent for the model
model Elastic_Inelastic_Power_Function
prop Transition_Force= set_Transition_force model_notension = 1 model_inheritprop =1
prop default_ks=9.0e6
;pause key
prop default_fric=0.6
prop Exponent = setExponent
set fishcall 6 catch_contact_model
;------------------------------------------------------------------
Cycle 90000
;set plot avi size 640 480
;movie avi_open file Thesis_UDM_model.avi
;movie step 12 file Thesis_UDM_model.avi
cycle 30000
;movie avi_close file Thesis_UDM_model.avi
plot show
;=====================The End================================
```