1 Hypoplastic constitutive equation version von Wolffersdorff

The constitutive model version von Wolffersdorff [1] is as follows:

$$\overset{\circ}{\mathbf{T}} = f_b f_e \frac{1}{\mathrm{tr}\ (\hat{\mathbf{T}} \cdot \hat{\mathbf{T}})} \left\{ F^2 \mathbf{D} + a^2 \mathrm{tr}\ (\hat{\mathbf{T}}\mathbf{D})\hat{\mathbf{T}} + f_d a F \left[\hat{\mathbf{T}} + \hat{\mathbf{T}}^* \right] \|\mathbf{D}\| \right\} \tag{1}$$

Equation 1 can be written in incremental stress form as follows:

$$\Delta\sigma_{11} = f_b f_e \frac{1}{\mathrm{tr}\ (\hat{\sigma} \cdot \hat{\sigma})} \left\{ F^2 \Delta\varepsilon_{11} + a^2 \mathrm{tr}\ (\hat{\sigma}\Delta\varepsilon)\hat{\sigma}_{11} + f_d a F \left[\hat{\sigma}_{11} + \hat{\sigma}^*_{11} \right] \|\Delta\varepsilon\| \right\} \tag{2}$$

$$\Delta\sigma_{22} = f_b f_e \frac{1}{\mathrm{tr}\ (\hat{\sigma} \cdot \hat{\sigma})} \left\{ F^2 \Delta\varepsilon_{22} + a^2 \mathrm{tr}\ (\hat{\sigma}\Delta\varepsilon)\hat{\sigma}_{22} + f_d a F \left[\hat{\sigma}_{22} + \hat{\sigma}^*_{22} \right] \|\Delta\varepsilon\| \right\} \tag{3}$$

$$\Delta\sigma_{33} = f_b f_e \frac{1}{\mathrm{tr}\ (\hat{\sigma} \cdot \hat{\sigma})} \left\{ F^2 \Delta\varepsilon_{33} + a^2 \mathrm{tr}\ (\hat{\sigma}\Delta\varepsilon)\hat{\sigma}_{33} + f_d a F \left[\hat{\sigma}_{33} + \hat{\sigma}^*_{33} \right] \|\Delta\varepsilon\| \right\} \tag{4}$$

$$\Delta\sigma_{12} = f_b f_e \frac{1}{\mathrm{tr}\ (\hat{\sigma} \cdot \hat{\sigma})} \left\{ F^2 \Delta\varepsilon_{12} + a^2 \mathrm{tr}\ (\hat{\sigma}\Delta\varepsilon)\hat{\sigma}_{12} + f_d a F \left[\hat{\sigma}_{12} + \hat{\sigma}^*_{12} \right] \|\Delta\varepsilon\| \right\} \tag{5}$$

With:

$$a = \frac{\sqrt{3}(3 - \sin\varphi_c)}{2\sqrt{2}\sin\varphi_c} \tag{6}$$

$$F = \sqrt{\frac{1}{8}\tan^2\psi + \frac{2 - \tan^2\psi}{2 + \sqrt{2}\tan\psi\cos 3\vartheta}} - \frac{1}{2\sqrt{2}}\tan\psi \tag{7}$$

$$\tan\psi = \sqrt{3}\,\|\hat{\sigma}^*\| \tag{8}$$

$$\cos 3\vartheta = -\sqrt{6}\frac{\mathrm{tr}\ (\hat{\sigma}^* \cdot \hat{\sigma}^* \cdot \hat{\sigma}^*)}{[\mathrm{tr}\ (\hat{\sigma}^* \cdot \hat{\sigma}^*)]^{3/2}} \tag{9}$$

The characteristic void ratio are:

$$\frac{e_i}{e_{i0}} = \frac{e_c}{e_{c0}} = \frac{e_d}{e_{d0}} = \exp\left[-\left(-\frac{\text{tr }\sigma}{h_s}\right)^n\right] \tag{10}$$

The pyknotropy function are:

$$f_e = \left(\frac{e_c}{e}\right)^\beta \tag{11}$$

$$f_d = \left(\frac{e - e_d}{e_c - e_d}\right)^\alpha \tag{12}$$

And the barotropy function is:

$$f_b = \left(\frac{h_s}{n}\right)\left(\frac{1+e_i}{e_i}\right)\left(\frac{e_{i0}}{e_{c0}}\right)^\beta \left(-\frac{\text{tr }(\sigma)}{h_s}\right)^{1-n} \left[3 + a^2 - \sqrt{3}a\left(\frac{e_{i0} - e_{d0}}{e_{c0} - e_{d0}}\right)^\alpha\right]^{-1} \tag{13}$$

with:

$$\hat{\sigma} = \frac{\sigma}{\text{tr}\sigma} \tag{14}$$

$$\hat{\sigma}^* = \hat{\sigma} - \frac{1}{3}\mathbf{I} \tag{15}$$

and void ratio increment is calculated as follows:

$$\Delta e = (1+e)(\Delta\varepsilon_{11} + \Delta\varepsilon_{22} + \Delta\varepsilon_{33}) \tag{16}$$

Where:
h_s = granular stiffness
n = exponent of comprssion law
φ_c = critial friction angle
e_{c0} = critical void ratio fro sig=0
e_{d0} = void ratio at maximum density for sig=0
e_{i0} = void ratio at minimum density for sig=0
α = pycnotropy exponent
β = pycnotropy exponent

References

[1] von Wolffersdorff, P.-A., A hypoplastic relation for granular materials with a predefined limit state surface, Mechanics of Cohesive-Frictional Materials, vol. 1, 251-271 (1996)

[2] Marcher Th., Vermeer P.A and von Wolffersdorff P.-A., Hypoplastic and elastoplastic modelling -a comparison with test data, Constitutive modelling of granullar materials, Springer-verlag Berlin Heidelberg (2000)